Developmental Differences in Young Children's Solutions of Logical vs. Empirical Problems

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Abstract

We examined the development of the ability to differentiate logically determinate from logically indeterminate problems. The results indicated that a) young children tend to reduce the number of empirical possibilities via "cutting" the second half of less informative propositions, b) these errors do not stem from encoding or recall errors, c) from elementary to middle school, children tend to increase their understanding of logical form, and d) this increase corresponds to a decrease in the rate of cuts.

There is a large body of research examining children's understanding of empirical indeterminacy (Fay & Klahr, 1996; Piéraut-Le Bonniec, 1980; Sodian, Zaitchik, & Carey, 1991). A problem is empirically determinate if it corresponds to exactly one empirical possibility; otherwise, it is empirically indeterminate (Piéraut-Le Bonniec, 1980). Previous findings suggest that young children often (1) fail to appreciate empirical indeterminacy, confusing indeterminate problems with determinate ones, but not vice versa, and (2) have less difficulty solving determinate problems than indeterminate problems (Bindra, Clarke, & Schultz, 1980; Byrnes & Overton, 1986; Fay & Klahr, 1996; Pieraut-Le Bonniec, 1980). However, there is another class of problems that should be considered in conjunction with the issue of determinacy- problems that require logical, but not empirical solutions. These problems are logically determinate (LD) if they are solved logically, but they are indeterminate if they are solved empirically.

Researchers have demonstrated that children (and many adolescents) do not fully understand logical determinacy (Byrnes & Overton, 1986; Moshman & Franks, 1986; A. Morris & Sloutsky, 1998) and they often attempt to provide empirical solutions to logically determinate problems (A. Morris & Sloutsky, 1998; Osherson & Markman, 1975). In this article, we examine the development of solution strategies to some logical and empirical problems and possible cognitive mechanisms underlying these strategies.

Information-processing analysis of solving logical vs. empirical problems

Logically determinate problems are those that can be solved *a priori* based on their logical form. Some LD problems yield logically true or necessary conclusions, whereas others yield logically false or impossible conclusions. Problems that yield conclusions that are true in some, but not all states of affairs are defined as logically indeterminate (LI), or empirical.

It has been traditional since the early work of Newell & Simon (1972) to conceptually model problem solving as search through a problem space for a desired goal state. A three-stage model outlines the creation of problem space, search and creating an output. Encoding is the creation of problem space from the information in the environment. The more clearly the problem space represents salient elements in the environment, the more veridical the representation, and the greater opportunity the organism has for solving the problem (Newell & Simon, 1972; Newell, 1990). The second phase is search in which a decision matrix is examined for possible outcomes and actions of the represented problem. Two types of search are utilized: problem search, in which search proceeds through possible outcomes of states and operators, and knowledge search, in which search is through memory (Newell, 1990). Once a goal state, an impasse, or some terminating point in search is reached, an output is then created. Therefore, it seems safe to assume that the mentioned reasoning and problem solving errors may occur due to the following factors (or any of their combinations):

1) Limits on encoding.
2) Poor representation of problem space.
3) Incomplete search through problem or memory space.
4) Inaccurate mapping of a problem solution onto a verbal response.

Evidence from several related domains such as scientific reasoning, logical reasoning, and practical reasoning indicates that children and adults often limit their search in both problem space and memory space (Kuhn et al., 1995; Markovits, 1988; Mynatt, Dohetry, &

Tweney, 1977; Tweney et al, 1980). They were also found to exhibit a 'positive capture' strategy failing to consider equally plausible alternatives (Bindra et al., 1980; Fay & Klahr, 1996). The tendency to limit their search has also been found in studies of logical reasoning in that both children and adults tend to not search for counterexamples in forms such as conditional reasoning (Johnson-Laird, 1993; Markovits, 1988; Wason & Johnson-Laird, 1972). In practical reasoning (when the task was to compare size of foreign cities) people often made fast and frugal "Take-the-Best" inferences relying on a small number of the most salient cues (such as the familiarity of the city), while ignoring the rest of the cues (Gigerenzer & Goldstein, 1996).

There is also evidence that young children exhibit difficulties when solving problems corresponding to no empirical possibilities, such as contradictions (B. Morris, 1998; Sloutsky, Rader, & Morris, 1998). In particular, when presented with a contradiction, preschoolers responded as if they ignored (or "cut") the second half of the contradiction thus transforming it into a statement corresponding to one empirical possibility functioning to limit problem space.

Therefore, it seems plausible that young children implicitly assume that propositions correspond to exactly one empirical possibility, thus creating a "defective" or incomplete problem space. If this is the case, then the number of empirical possibilities compatible with the problem could predict the problem difficulty. The easiest problems are those that correspond to exactly one empirical possibility whereas an increase or decrease in the number of empirical possibilities leads to an increase in problem difficulty and subsequently to the number of errors (for a detailed discussion see Sloutsky, Morris, & Rader, in review).

Three groups of children (preschool, elementary and middle school) were presented with reasoning problems. The experiment focused on (a) the ability of children and adolescents to distinguish logical from empirical problems; (b) solution strategies for different types of problems; (c) patterns of errors; (d) the relationship between the number of empirical possibilities and the problem difficulty; and (e) accuracy of encoding and mapping of verbal responses. We deemed it necessary to reduce the number of possible sources of error via eliminating the necessity of search through knowledge space. In so doing, we presented participants with knowledge-lean problems that required "deriving a solution from givens" rather than requiring memory search.

Method

Participants

38 four-and five- year-old children enrolled in three child care centers (average age = 4.3 years; 16 girls and 22 boys), 34 third grade children in three elementary school classrooms (average age =8.4 years; 19 girls and 15 boys), and 35 sixth grade children enrolled in two middle school classrooms (average age = 11.7 years, 16 girls and 19 boys).

Materials

The tasks consisted of a series of predictions by an imaginary character, ZZ, as to the outcome of one of two separate items a) a ball dropped in the Tautology Machine and b) opening or not opening a book. The Tautology machine is a 21" x 24" board with a chute at the top in which a ball dropped will fall to one of two terminating points (in this experiment labeled "Red" and "Green"). The "predictions" took the form of one of four logical forms Tautologies, Contradictions, Disjunctions, and Conjunctions. The Tautology Machine has a switch (occluded from participants) in the back of the machine that moves a lever to one of two sides directing the ball to either the red or the green side. ZZ made eight predictions regarding the outcomes of the game, two predictions for each logical form. The experimenter presented ZZ, the Tautology Machine that has two possible outcomes, and a book that could be either opened or closed. ZZ made predictions pertaining (1) only to the ball's landings (tautologies and contradictions) and (2) to the ball's landing and to whether the book will be opened or closed (conjunctions and disjunctions).

Procedure

In this experiment, there was one within-subject factor, the logical form of the prediction. The experiment was conducted in a single 10-15 minute session that included two phases: warm-up/instruction phase and the experimental phase. In the warm-up phase, each child was read a set of instructions that explained the purpose of the game as evaluating the predications of an imaginary character named "ZZ." Six participants were eliminated from further consideration because they gave "Can't tell" responses to all warm-up questions.

Participants were asked five questions about each prediction. (1) An encoding measure (repeat the prediction). (2) An *a priori* evaluation, of the prediction ("True," "False," and "Can't tell). (3) Request for empirical verification ("Do we need to drop the ball?"). (4) An *a posteriori* evaluation of the prediction (only if empirical verification was requested). And (5) a measure of encoding (repeat the initial prediction). The recall measure was introduced only with elementary school and middle school.

Results

A priori evaluation

A two way 3 (age group) by 4 (logical form) repeated measures ANOVA was performed on the a priori evaluations with age as a between-participant factor and logical form as a repeated measure. The analysis yielded a significant main effect for age, F (2, 98)= 28.890, p<.001, form, F (2,98)= 5.3171, p<.002, and the interaction between age and logical form, F (2, 98)= 3.171, p<.005. Tukey's HSD post-hoc tests indicated that for conjunctions and tautologies, middle school children made more correct a priori evaluations than both elementary and preschool children, Tukey's HSD, all ps< .01. For contradictions, middle school made more correct a priori evaluations than elementary school children, Tukey's HSD, p< .01. Middle school children made more correct a priori evaluations of disjunctions than preschool children, Tukey's HSD, p< .02. Post-hoc tests by form indicate that conjunctions and disjunctions were evaluated as correct more frequently than tautologies and contradictions more frequently than tautologies , Tukey's HSD, p< .02.

Requests for Empirical Verification

To analyze requests for empirical verification, participants' responses to particular problems were collapsed into two groups, responses to logically determinate problems (i.e., tautologies and contradictions) and to logically indeterminate problems (conjunctions and disjunctions). These collapsed responses were subjected to two-way 3 (age group) by 2 (logical determinacy v. logical indeterminacy) repeated measures ANOVA. The analysis yielded a main effect for age, F (2, 98)= 75.974, p< .0001, form, F (2,98) = 51.061, p< .0001, and the interaction between age and logical forms, F (2, 98)= 44.545, p<.0001. For logically determinate forms, middle school children requested significantly less empirical verification than elementary and preschool children, Tukey's HSD, ps< .001. For logically indeterminate forms, middle school children requested more empirical verification than elementary and preschool children, Tukey's HSD, ps< .01. Additionally, elementary school children requested less empirical verification than preschool children, Tukey's HSD, ps< .01. While younger children equally frequently requested empirical verifications for both logically determinate and logically indeterminate forms, older children less frequently requested empirical verification of logically determinate forms. Within-age group differences were only present in middle school children on logically determinate forms, Tukey's HSD, p< .001, all others were not significantly different.

Cuts

As stated earlier, we hypothesized that the number of empirical possibilities compatible with a proposition will be a predictor of accuracy of processing. It is also predicted that those children who did not recognize logical form should exhibit more distortions of data as a function of the predicted informativeness of each proposition. Furthermore, we expected these distortions to exhibit systematicity. The pattern to be analyzed is a "cut" that functions to reduce the number of empirical possibilities. To analyze patterns of errors, a rigorous decision procedure was introduced into the analysis to distinguish "cuts" from logically appropriate responses in which the second half of propositions was "rigged" so that "cuts" could be distinguished form other responses.

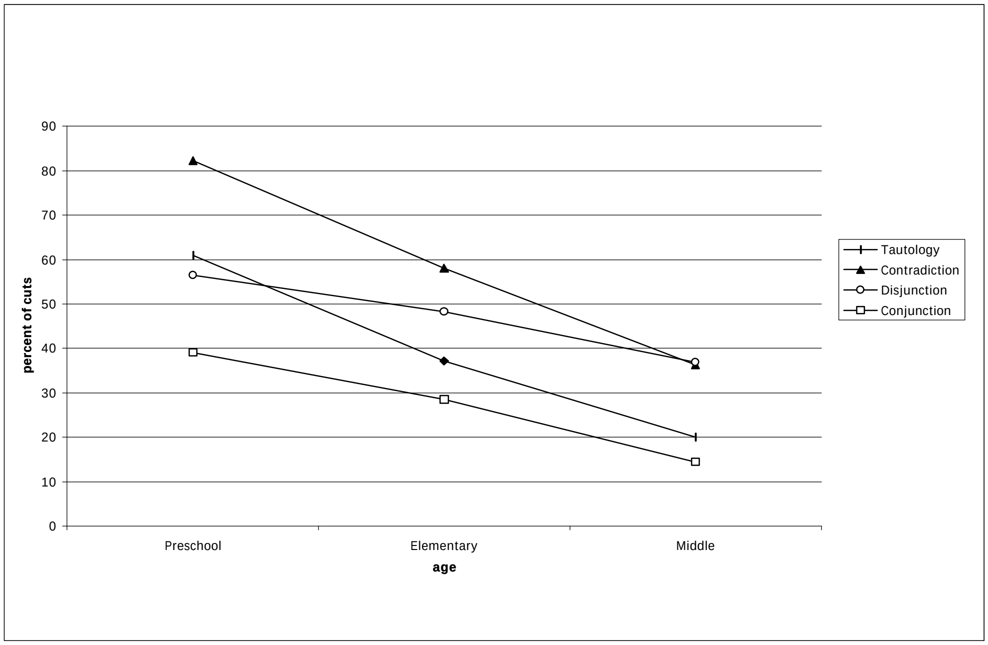

Figure 1- Percent of "cuts" of logical propositions by age and logical form

A series of McNemar and Cochran's chi-squares were conducted to compare conversion levels by logical form. The error rates are depicted in Figure 2. Data in the figure suggest that in the younger groups, conjunctions were least likely to be cut, whereas contradictions and tautologies were most likely to be cut, and disjunctions were in-between the two extremes. In middle school children, disjunctions were cut at the highest rate while low levels of conversions of tautologies and contradictions seem to be due to recognition of logical form and therefore eliminating the need for empirical evidence and its effect on outcome. For preschool age children, a Cochran *Q* test, with McNemar chi-square tests for post hoc pairwise comparisons, indicated significant differences in conversion rates across the logical forms (Cochran *Q* (3, 32) = 22.3, p<. 0001). Pairwise comparisons indicate that conjunctions were significantly less probable to be converted than disjunctions (*McNemar* (1, 37) = 4.5, p <. 05), tautologies (*McNemar* (1, 45) = 14.7, p <. 0001), and contradictions (*McNemar* (1, 46) = 28, p <. 0001), whereas disjunctions are less probable to be converted than contradictions (*McNemar* (1, 36) = 10.3, p <. 005). These data suggest that the probability of a conversion increases with an increase in the number of possibilities compatible with the statement.

In elementary school children, a Cochran *Q* test, with McNemar chi-square tests for post hoc pairwise comparisons, indicated significant differences in conversion rates across the logical forms (Cochran *Q* (3, 34= 21.7, p<. 0001). Pairwise comparisons indicate that conjunctions were significantly less probable to be converted than contradictions (*McNemar* (1, 35) = 9.62, p <. 001), whereas contradictions are more probable to be converted than disjunctions (*McNemar* (1, 35) = 6.0, p <. 025), and tautologies (McNemar (1, 35= 8.9, p<.005). These data, like preschool data, suggest that the probability of a conversion increases with an increase in the number of possibilities compatible with the statement.

In middle school children, a Cochran *Q* test, with McNemar chi-square tests for post hoc pairwise comparisons, indicated significant differences in conversion rates across the logical forms (Cochran *Q* (3, 35) = 24.6, p<. 0001). Pairwise comparisons indicate that conjunctions were significantly less probable to be converted than contradictions (*McNemar* (1, 34) = 6.36, p<. 025), and disjunctions (*McNemar* (1, 34) = 15.96, p <. 001), whereas disjunctions are more probable to be converted than tautologies (McNemar (1,34)= 6.0, p<.025) and conjunctions (*McNemar* (1, 34) = 9.45, p <. 005). These data indicate that for all age groups, problems corresponding to exactly one empirical possibility (i.e., conjunctions) elicit fewer errors than problems corresponding to more than one or less than one empirical possibility. Data in the figure, also indicate a marked developmental progression with respect to error rates.

Accuracy of encoding & recall

From the very beginning, we have contemplated several possibilities as to where in the course of information processing conversions take place. Two possibilities seemed most plausible: (1) conversions occur in the course of encoding the propositions into working memory or (2) in the course of retrieving the propositions from long-term memory, or (3) they occur in the course of the creation of a problem space.

The results indicate that preschool children encoded about 23% of all predictions incorrectly while elementary and middle school children encoded less than 10% incorrectly. The data demonstrate that the levels of encoding errors cannot account for conversions for two reasons: a) mean encoding levels would have to increase dramatically to equal those levels in conversions and b) the rates of encoding errors are lowest for those forms which are converted at the highest rates. Therefore, the evidence does not suggest that encoding is responsible for conversions.

Recall rates indicate that overall children tend to recall the initial predictions correctly, even those that had high conversion rates. These findings seem to suggest that participants represent a proposition veridically without veridically representing corresponding state of affairs. This dissociation indicates that the reported conversions do not stem from memory limitations. It seems that a likely source of these conversions is an incomplete representation of empirical possibilities or states of affairs compatible with the proposition.

Discussion

The findings could be summarized as follows. (1) As opposed to middle school children, young children do not distinguish between logical and empirical problems, and often attempt to empirically solve logical problems. (2) Participants of all age groups who exhibited errors, exhibited the same pattern of errors — the tendency to represent a problem as if it were compatible with only one empirical possibility. (3) In so doing, across the age groups, participants exhibited one strategy- a “cut” in the second half of logical propositions. (4) Cuts markedly decreased with age and the acquisition of an understanding of logical form.

As in previous experiments (Fay & Klahr, 1996; Sloutsky, Morris, & Rader, in review) preschool and elementary children did not distinguish logically determinate from empirical statements. Two sets of evidence demonstrate a lack of recognition of logical sufficiency of tautologies and contradictions in preschool and elementary school children; (1) no recognition of *a priori* logical form and (2) requests for empirical verification. This evidence also suggests that middle school age children differentiate logically determinate from indeterminate problems.

Cuts were demonstrated by *a posteriori* evaluations of the ball’s landing that systematically ignored the second half of the proposition in question. For example, when given “the ball will land on red and not red” and the ball actually landed on red, the child responds that the prediction was correct. Two factors are necessary in order to draw this conclusion: a) inability to distinguish *a priori* logical form, and b) inability to recognize when empirical verification is necessary.

The data also indicate that problems compatible with more than one or less than one empirical possibility elicited more errors than problems compatible with exactly one empirical possibility. Furthermore, participants of all age groups exhibited the same pattern of errors — a “cut” in the second half of propositions.

Developmental trends suggest that the proportion of “cuts” decreases with the acquisition of an understanding of logical form, theoretically a more adaptive strategy for solving these types of problems. While preschool and elementary school children did not recognize a priori logical form and did not differentiate determinate from indeterminate problems in terms of empirical verification, middle school children performed significantly better on both measures. Additionally, levels of cuts were very low for middle school children overall. Encoding and recall rates were not significantly different for all groups and did not occur at levels high enough to account for conversion phenomena. Therefore, developmental changes that seem to be related to the decrease in cuts are an increase in recognition of logical forms and a decrease in requests for empirical verification in logically determinate forms. These changes suggest the acquisition of a strategy of logical reasoning would also function to supplant search through problem space for a solution with a search through knowledge space for the correct solution. This suggests that as children acquire an understanding of logically determinate problems the need for limiting the number of possibilities compatible with the problem decreases.

Conclusion

The presented evidence supports the hypothesis that problems compatible with more than one possibility or less than one possibility elicit more errors than problems compatible with exactly one possibility. Three main findings seem to be particularly important: (a) as opposed to middle school children, preschoolers and elementary school children do not distinguish logically determinate from logically indeterminate forms; (b) cuts seem to stem from the creation of an incomplete problem space, and not from inaccuracies in encoding; and (c) decrease in the rate of cuts corresponds to increases in the ability to distinguish logical from empirical problems. However, additional studies will help establish what else is missing and what else develops in solving logical versus empirical problems.

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